On the Dynamic Effect of Infinite Redshift Surface: Lorentz Factor Analysis Method

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Abstract

The generation of infinite redshift surface is due to the singularity of the Lorentz transformation; according to this fact, analyzing the singularity of the parameters in the Lorentz transform factor, which is named the Lorentz factor analysis method in this paper, can be used to research the dynamic effect of the infinite redshift surface. In this paper, the method is applied to the Reissner-Nordstrom and Kerr-Newman black hole spacetime, which finds that the infinite redshift surface will expand and contract with the observer's radial motion, and the corresponding equations are given clearly. A new proof of the inaccessibility of the time-like singularity in a charged black hole is given, and the inaccessibility of the time-like singularity is proved only related to the electrical nature, it has nothing to do with rotation. The study result of space-like quasi-circular motion observer shows that the infinite redshift surface is consistent with that of the static observer at infinity in this case. The minimum blueshift radius which is proposed in the study of the observer who escapes at the speed of light in normal spacetime, it makes a stronger constraint on black hole parameters than traditional condition. This is a possible reason why it is difficult to form an intermediate-mass black hole; discussion about it can be found at the end of the paper.

Keywords: Dynamic effect, Infinite redshift surface, Black hole, Time-like singularity

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1. Introduction

At present, the research on the event horizon is rich, but the infinite redshift surface is not so much[1-4]. We know that the horizon has many thermodynamic and quantum mechanical properties[5-6], and the infinite redshift surface is of great significance in practical observation because it is closely related to redshift and blueshift, so it is important too. The Lorentz transformation is an important foundation in general relativity; it gives the correspondence of physical law between different reference systems. But the transformation is parameter dependent. For example, in Minkowski spacetime of special relativity, the parameter is expressed as relative velocity, and this leads to the singularity of Lorentz transformation at some parameter points. But it doesn't represent the real singularity of spacetime; the curvature scalar formed by the Ricci and Riemannian curvature tensor at the singular points is probably not divergent[7]. This shows that the singularity of Lorentz transformation corresponds to the infinite redshift surface, which might be caused by the singularity of coordinates. From this point of view, we can study the variation of the singularity of Lorentz transform with parameters in various backgrounds of spacetime, to get the dynamic effect of the infinite redshift surface. We first discuss the Reissner-Nordstrom black hole spacetime as the most general spherically symmetric spacetime, because of the symmetry requirement, we only need to discuss the radial motion. Secondly, the Kerr-Newman black hole spacetime is studied as the most general spacetime in classical general relativity, while we need to study not only the radial motion but also the angular motion. Besides, we are also interested in another problem, that is, in normal spacetime, what size is the infinite redshift surface look like when escaping at the speed of light?

This paper studies these problems and some related things. For convenience, take metric sign as $-+++$, and using the natural unit system, $G=c=1$.

2. Lorentz factor analysis method

The analysis of various properties of infinite redshift surfaces using the Lorentz factor in curved spacetime has not been proposed systematically before. The metric of Minkowski spacetime is simple and very basic; the analysis method can be made clear by starting from it.

Specifically, we need to change the coordinate of motion to the time coordinate, such as replacing $\mathrm{d}r$ with $u\,\mathrm{d}t$ and $\mathrm{d}\theta$ with $\dot{\theta}\,\mathrm{d}t$, where u is the radial velocity and $\dot{\theta}$ is the angular velocity. In this way, the singularity of the metric can be expressed uniformly in the form of the time coordinate, as we did in the Lorentz transformation.

Consider a flat metric

$$\mathrm{d}s^2=-\mathrm{d}t^2+\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2=-(1-u^2)\mathrm{d}t^2 \tag{1}$$

The meaning of the second equal sign is to extract the velocity parameter. Equation (1) is the observation of the static observer, compare it with the result of the follow-up observer ($\mathrm{d}s^2=-\mathrm{d}\tau^2$, $\mathrm{d}\tau$ is the proper time), and we can get the Lorentz transform of the time

$$\mathrm{d}t=\frac{\mathrm{d}\tau}{\sqrt{1-u^2}} \tag{2}$$

We are used to using γ symbol to represent the factor of the Lorentz transformation. In the example above, $\gamma=1/\sqrt{1-u^2}$. The Lorentz transformation includes the existence of γ, which means the square root contained in γ must be meaningful, and γ itself should be meaningful too. Later, we will show how to use this property to analyze the infinite redshift surface.

The infinite redshift phenomenon can be attributed to the proper time is zero on a hypersurface in spacetime, from the perspective of the Lorentz transformation, the transformation between the static observer at infinity and the follow-up observer on the hypersurface, and the Lorentz factor γ at a singular point. If we think of the parameter u as the velocity on the hypersurface of the spacetime manifold, the Lorentz factor γ will give that u cannot be the speed of light $c=1$. This shows that the infinite redshift surface is indeed a null hypersurface.

For the velocity on hypersurface, we should consider it as the derivative of space-like coordinate to time-like coordinate. For example, in normal spacetime, $u=\mathrm{d}r/\mathrm{d}t$. Notice that the world line of an object with mass is time-like; there is $|u|\in[0,1)$. However, because of the exchange of space and time coordinates, the definition of velocity on a hypersurface in the one-way membrane region becomes $u=\mathrm{d}t/\mathrm{d}r$. Then, the velocity of an object with mass will be limited within $|u|\in(1,\infty)$. It should be noted that, in the one-way membrane region, the future direction of the light cone is unidirectional, and all physical processes are unidirectional, so the velocity parameter on the hypersurface exceeds the speed of light does not violate any laws of physics[8]. It should be noted that the definition of velocity here is at the size level, for the direction, which needs to be specifically pointed out in the specific problem. For example, in this paper, the displacement close to the essential singularity is considered positive, and the displacement far from the essential singularity is negative.

In addition, it can be seen from equation (2) that the Lorentz factor is a conversion factor for the time-like coordinates of the two measurement systems. So, the expression of the γ factor obtained by $\mathrm{d}t$ coordinate can only be applied to normal spacetime. Otherwise, the $\mathrm{d}t$ coordinates cannot be used, and the γ factor is obtained by using the time-like coordinates in the spacetime; for example, for the one-way membrane region of the Schwarzschild black hole, $\mathrm{d}r$ coordinate needs to be used. Due to the specificity of the one-way membrane region, we mainly discuss the observations in normal spacetime in the following, while the situation in the one-way membrane region is discussed separately in Section 6.

3. General spherically symmetric spacetime

According to the Birkhoff theorem, in general relativity, the most general spherically symmetric black hole is Reissner-Nordstrom (R-N) black hole[9]; we will take it as the spacetime background of this section.

Since it is a spherically symmetric spacetime, we are not interested in the motion around the black hole, so we only study the radial motion. Write directly the spacetime metric with velocity parameter u

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} u^2\mathrm{d}t^2 + \mathrm{d}\Omega^2 \tag{3}$$

where $\mathrm{d}\Omega^2 = r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)$ is a two-dimensional spherical hypersurface. r is the radial coordinate, M and Q are mass and charge, respectively.

We can calculate the Lorentz factor γ

$$\gamma = \sqrt{\frac{r^2(r^2 - 2Mr + Q^2)}{(r^2 - 2Mr + Q^2)^2 - u^2r^4}} \tag{4}$$

Let's analyze the singularity of equation (4). Because we want to research the dynamic effect of the infinite redshift surface, we need to exclude the infinite point $r = \infty$ and the essential singularity $r = 0$ first.

The zero points of the molecule, $r^2 - 2Mr + Q^2 = 0$, are the traditional inner and outer infinite redshift surface of the R-N black hole

$$r_\pm = M \pm \sqrt{M^2 - Q^2} \tag{5}$$

The results show that the infinite redshift surface at zero moving object velocity u coincides with the event horizon in static spacetime, this is related to the symmetry of spacetime; so equation (5) is also the event horizon of the R-N black hole.

The denominator has four zero points, which we denote as

$$\begin{aligned} r_1 &= \frac{M - \sqrt{M^2 - (1+u)Q^2}}{1+u} \qquad r_2 = \frac{M + \sqrt{M^2 - (1+u)Q^2}}{1+u} \\ r_3 &= \frac{M - \sqrt{M^2 - (1-u)Q^2}}{1-u} \qquad r_4 = \frac{M + \sqrt{M^2 - (1-u)Q^2}}{1-u} \end{aligned} \tag{6}$$

For convenience, we express them uniformly as r_i.

When $u = 0$, the singularity of equation (4) only depends on $r^2 - 2Mr + Q^2 = 0$; this result is consistent with the fact that the infinite redshift surface at zero velocity coincides with the horizons. When $u \neq 0$, the zero points of the denominator are r_i, not $r_\pm$, so equation (4) is singular at r_i, the two-dimensional spherical hypersurface at r_i, $\mathrm{d}\Omega|_{r_i}$, is the infinite redshift surface.

From equation (6), between r_1 and r_3, and between r_2 and r_4, can be corresponded to by changing the sign of u. Mathematically they are equivalent, but physically we can pick the correct dynamical surface for the infinite redshift surface by previous observations. Besides, in the past studies, we were generally required $M^2 > Q^2$ to avoid the negative energy problem, equation (6) extends this constraint to the case of the dynamics, giving a more general constraint $M^2 > (1+u)Q^2$.

We know that the infinite redshift surface is a coordinate singularity when the velocity of the object vanishes, and its spacetime curvature does not diverge. The effect of this kind of singularity only exists in the observation. So, r_i contains the rich properties related to the dynamic effect of the infinite redshift surface.

Firstly, according to astronomy, the redshift phenomenon resulting from the motion close to each other, and the blueshift phenomenon resulting from the motion far from each other, the motion of approaching a black hole from infinity implies the expansion of the outer infinite redshift surface. Because of this observational property, the outer infinite redshift surface, which is the observed outer surface of the black hole, should be chosen as r_4 at $u > 0$. As for the inner infinite redshift surface, since the observer is moving in normal spacetime within the inner infinite redshift surface, the case $u > 0$ implies that the observer is moving close to the singularity $r = 0$ and is moving away from the inner infinite redshift surface, and should undergo a blueshift phenomenon, the internal infinite redshift surface should shrink. In the range of $u \in (0, 1)$, r_3 satisfies the property.

The inner infinite redshift surface r_3 gives the property that the time-like singularity cannot be approached. There has been a lot of related works on the property that time-like singularity cannot be approached[10-12]. Because the region within the inner infinite redshift surface of the R-N black hole is a normal spacetime, that is, ∂_t is time-like, and ∂_r is space-like, the essential singularity $r = 0$ of the R-N black hole is a time-like singularity. In normal spacetime, $|u| \in [0, 1)$, to make the conclusion more significant, we take the limit case $u = 1$

$$r_3|_{u=1} = \lim_{u\to 1}\frac{M-\sqrt{M^2-(1-u)Q^2}}{1-u} = \frac{Q^2}{2M} \tag{7}$$

And the work of [10,11] proved that an object needs infinite acceleration to approach the time-like singularity, and if the inner horizon cannot be contracted to $r=0$, the particle cannot reach the time-like singularity. This is a very important result, it seems that the inner infinite redshift surface will contract as the observer moves, as long as the black hole is charged, $Q\neq 0$, the inner infinite redshift surface will never contract to the essential singularity. But if $Q=0$, the inner infinite redshift surface will shrink to the essential singularity, which is what happens in the Schwarzschild black hole. This result shows that it is impossible to approach the essential singularity in a charged black hole. The change rate of the inner infinite redshift surface with velocity u is given here

$$\frac{\mathrm{d}r_3}{\mathrm{d}u} = \frac{Q^2(1-u)-2M\left(M-\sqrt{M^2-(1-u)Q^2}\right)}{2\sqrt{M^2-(1-u)Q^2}\,(1-u)^2} \tag{8}$$

When Q and M are known, this rate is negative and increases with u.

Then, we should consider the expansion of the outer infinite redshift surface r_4. The change rate of r_4 with velocity is given

$$\frac{\mathrm{d}r_4}{\mathrm{d}u} = \frac{2M\left(M+\sqrt{M^2-(1-u)Q^2}\right)-Q^2(1-u)}{2\sqrt{M^2+(1-u)Q^2}\,(1-u)^2} \tag{9}$$

It can be proved numerically that, with the increase of u, the rate given by equation (8) is very small, while the rate given by equation (9) is surprisingly large, especially when u is close to 1. This is expected because the infinite redshift surface is a null hypersurface, the motion occurs on the null hypersurface when the velocity parameter u reaches 1, which can be understood as: when an object moves near the speed of light, expanding r_4 will "hit" it.

It is worth noting that, the infinite redshift surface is as objective as the horizon, any object has the horizon and infinite redshift surface. It's just that the ordinary object's mass is distributed outside the horizon, the density does not meet the condition for the formation of black holes. According to the results obtained above, when two objects move relative to each other, the expansion of the outer infinite redshift surface will occur, thus changing the observational property of spacetime. This is probably the essence of the redshift phenomenon. We can calculate the relative redshift by calculating the expansion degree of the outer infinite redshift surface.

The opposite is the blueshift phenomenon. When we take the velocity parameter u in r_4 as a negative value (which means moving away from $r=0$), r_4 contracts as u getting more negative. Since all kinds of spacetime discussed in 4-D general relativity are asymptotically flat, at most, we can escape at the speed of light and r_4 will shrink to

$$r_{mb} = \frac{1}{2}\left(M+\sqrt{M^2-2Q^2}\right) \tag{10}$$

I call it the Minimum Blueshift Radius, it is the smallest observation size of a black hole when we escape from a black hole in an asymptotically flat spacetime. In Schwarzschild, $Q=0$, equation (10) gives $r_{mb}=M=R_s/2$, where $R_s=2M$ is the Schwarzschild radius. For more details on it, please see Section 5.

It is worth noting that, based on the discussion of the redshift and blueshift phenomena above, all surfaces mentioned in equation (6) are meaningful.

For the case of $u>0$, if the motion occurs outside the outer infinite redshift surface, both the inner and outer infinite redshift surface will expand, the outer infinite redshift surface is described by r_4 and the inner infinite redshift surface is described by r_1; if the motion occurs within the inner infinite redshift surface, both the inner and outer infinite redshift surface will shrink, the outer infinite redshift surface is described by r_2 and the inner infinite redshift surface is described by r_3.

Since r_4 and r_2, r_3 and r_1, have symmetry about u (Interconversion under the substitution of $-u$ and u), the dynamical surfaces describing the inner and outer infinite redshift surface are the same for $u<0$ as for $u>0$.

The following diagram illustrates the variation of the inner and outer infinite redshift surfaces with u

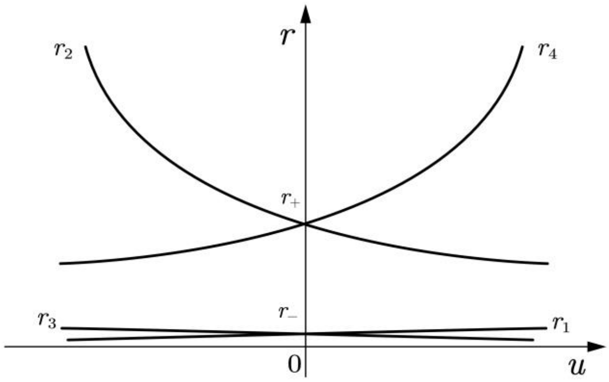

Figure 1. Schematic diagram of the variation of the inner and outer infinite redshift surfaces with the observer's movement velocity

In the diagram, r_- and r_+ represent the inner and outer infinite redshift surfaces under stationary observation (i.e., conventional), respectively.

4. Steady-state axisymmetric spacetime

We continue to use the Lorentz factor analysis method to research the effect of the infinite redshift surface. This time, we discuss the Kerr-Newman black hole spacetime as the most general black hole in classical general relativity. Let's first write down the metric of the Kerr-Newman black hole[13]

$$\begin{aligned} \mathrm{d}s^2 = &-\left(1-\frac{2Mr-Q^2}{\rho^2}\right)\mathrm{d}t^2+\frac{\rho^2}{\Delta}\mathrm{d}r^2+\rho^2\mathrm{d}\theta^2 \\ &+\left[(r^2+a^2)\sin^2\theta+\frac{(2Mr-Q^2)a^2\sin^4\theta}{\rho^2}\right]\mathrm{d}\varphi^2-\frac{2(2Mr-Q^2)a\sin^2\theta}{\rho^2}\mathrm{d}t\,\mathrm{d}\varphi \end{aligned} \tag{11}$$

Where $\rho^2=r^2+a^2\cos^2\theta$, $\Delta=r^2-2Mr+a^2+Q^2$ and $a=J/M$ means angular momentum per unit mass.

According to the symmetry of spacetime, we do not intend to study the rotation related to φ. So there are two velocity parameters left: $u=\mathrm{d}r/\mathrm{d}t$ and $\omega=\mathrm{d}\theta/\mathrm{d}t$, we first study the effect of u, then we should consider $\theta-\varphi$ curved surface as a two-dimensional space-like hypersurface. The g_{00} component and Lorentz factor of metric (11) is

$$g_{00}=-1+\frac{2Mr-Q^2}{r^2+a^2\cos^2\theta}+\frac{(r^2+a^2\cos^2\theta)u^2}{r^2-2Mr+a^2+Q^2}=-\gamma^{-2} \tag{12}$$

Because the formula of γ is a little complicated to write, to take care of the layout, we don't write it in detail anymore, you can calculate by yourself. Similar to Section 3, the zero points of the molecule gives the inner and outer event horizon of the Kerr-Newman black hole

$$\begin{gathered} (r^2+a^2\cos^2\theta)\,(r^2-2Mr+a^2+Q^2)=0 \\ r_\pm=M\pm\sqrt{M^2-a^2-Q^2} \end{gathered} \tag{13}$$

And zero points of the denominator have very complex expressions, but according to the fundamental theorem of algebra, we can still judge that this fourth ordered algebraic equation has four roots. We can study two special points of θ to avoid complexity, the north pole $\theta=0$, and the equatorial plane $\theta=\pi/2$. The zero points of the denominator at the north pole are

$$
\begin{aligned}
r_1 &= \frac{M-\sqrt{M^2-(1-u)\left[(1-u)a^2+Q^2\right]}}{1-u} \qquad r_2 = \frac{M+\sqrt{M^2-(1-u)\left[(1-u)a^2+Q^2\right]}}{1-u} \\
r_3 &= \frac{M-\sqrt{M^2-(1+u)\left[(1+u)a^2+Q^2\right]}}{1+u} \qquad r_4 = \frac{M+\sqrt{M^2-(1+u)\left[(1+u)a^2+Q^2\right]}}{1+u}
\end{aligned} \tag{14}
$$

According to the discussion of dynamical infinite redshift surfaces in Section 3, we can analyze the specific surface based on a specific motion.

Because of the characteristic of φ direction rotation, spacetime does not rotate at the north pole $\theta=0$, everything is similar to the R-N black hole. That means the infinite redshift surface of the Kerr-Newman black hole coincides with the event horizon. The area within the inner horizon belongs to normal spacetime, so there should be $|u|\in[0,1)$. The essential singularity of the Kerr-Newman black hole is still time-like, similar to the R-N black hole spacetime, although from the perspective of the manifold, it is no longer a point but a ring. Next, we will prove that even if the singular ring of the Kerr-Newman black hole spacetime is located at $r=0,\ \theta=\pi/2$, it still cannot be approached from the $\theta=0$ direction. We can take the limit case $u=1$ and consider the inner infinite redshift surface r_1

$$
r_1\big|_{u=1} = \lim_{u\to 1}\frac{M-\sqrt{M^2-(1-u)\left[(1-u)a^2+Q^2\right]}}{1-u} = \frac{Q^2}{2M} \tag{15}
$$

It is consistent with our result in the R-N black hole spacetime—it is obvious. But this result shows that the inaccessibility of singular ring is independent of θ, besides, the appearance of time-like singularity is only related to whether the black hole is charged, and has nothing to do with rotation. The effect of rotation is only to separate the infinite redshift surface from the event horizon.

The motion in the Kerr-Newman black hole spacetime also causes the expansion and contraction of the infinite redshift surface, corresponding to the redshift and blueshift phenomenon respectively. Then we can also study the minimum blueshift radius of the Kerr-Newman black hole spacetime, as we did in the previous section for the R-N black hole spacetime. From the asymptotically flat property, when the observer escape to infinity at the speed of light, r_2 will contract to

$$
r_{mb}\big|_{\theta=0} = \frac{1}{2}\left(M+\sqrt{M^2-2Q^2-4a^2}\right) \tag{16}
$$

When $a=0$, everything goes back to R-N black hole. Here we can see that the minimum blueshift radius is related to the three main parameters of the black hole (angular momentum, mass, and charge), and its attenuation is determined by charge and angular momentum, the weight of angular momentum per unit mass is twice that of charge.

So far, everything seems to be going well, but the discussion on the equatorial plane is not so concise. When $\theta=\pi/2$, the expression of zero points is too complex. However, with the help of mathematical software such as Mathematica, we can take some numerical techniques to study the problems of interest. The inner and outer infinite redshift surfaces on the equatorial plane are denoted as r_5 and r_6 respectively.

For whether the singular ring can approach from the $\theta=\pi/2$ direction. Of course, when $a=0$, there is the above conclusion $r_1\big|_{u=1}^{a=0}=Q^2/2M$, this result can be an important boundary condition. We can take the normal spacetime limit case $u=1$, and get $r_5\big|_{u\to 1}^{Q\to 0}=0$. If Q doesn't go to zero, $r_5\big|_{u\to 1}^{Q\to 0}\neq 0$. This shows again that the inaccessibility of time-like singularity is only related to the electrical nature of the black hole.

For the minimum blueshift radius, through numerical calculation, we can know that $r_{mb}\big|_{\theta=\pi/2}$ must be larger than $r_{mb}\big|_{\theta=0}$. That is to say, the infinite redshift surface of the Kerr-Newman black hole is an ellipsoid, and this geometric characteristic will not change with the motion of the observer. But its size in the eyes of the moving observer has changed compared with that of the static observer at infinity.

It should be noted that you may guess the expression of the minimum blueshift radius of the equatorial plane based on the traditional outer infinite redshift surface of the Kerr-Newman black hole on the equatorial plane, $r_+=M+\sqrt{M^2-Q^2}$. But the research result shows that

$$
r_{mb}\big|_{\theta=\pi/2} \neq \frac{1}{2}\left(M+\sqrt{M^2-2Q^2}\right) \tag{17}
$$

Its specific expression can be calculated by the software，but it is very complex.

Due to the spacetime symmetry, we can also discuss the effect of the infinite redshift surface in the θ direction. This time, the g_{00} component and Lorentz factor of metric (11) is

$$g_{00} = -1 + \frac{2Mr - Q^2}{r^2 + a^2\cos^2\theta} + (r^2 + a^2\cos^2\theta)\omega^2 = -\gamma^{-2} \tag{18}$$

According to the definition of generalized radius in elliptic coordinates $\rho^2 = r^2 + a^2\cos^2\theta$, the molecule of the Lorentz factor has no zero points. That is, we just need to analyze the zero points of the denominator.

The zero points of the denominator corresponding to two infinite redshift surfaces, however, because equation (18) involves the operation of trigonometric function power, the analytical formula of two infinite redshift surfaces is very complicated. We can decompose the tangent velocity in the $\theta = 0$ and $\theta = \pi/2$ directions, and then use the radial conclusions to study the problem of rotation.

We know that there is a relationship between the tangential velocity and the angular velocity as $v = \omega r$, if $|v| \in [0, 1)$ in normal spacetime, there should be $\omega = v/r$. It shows that discussing the motion in the θ direction at infinity is meaningless, there is no minimum blueshift radius in this case. Decompose the velocity into two directions, there is $u_y = v\sin\theta = \omega r\sin\theta$ in the $\theta = 0$ direction, and $u_x = v\cos\theta = \omega r\cos\theta$ in the $\theta = \pi/2$ direction.

Let's start with a qualitative discussion. Here is the profile of the Kerr-Newman black hole spacetime

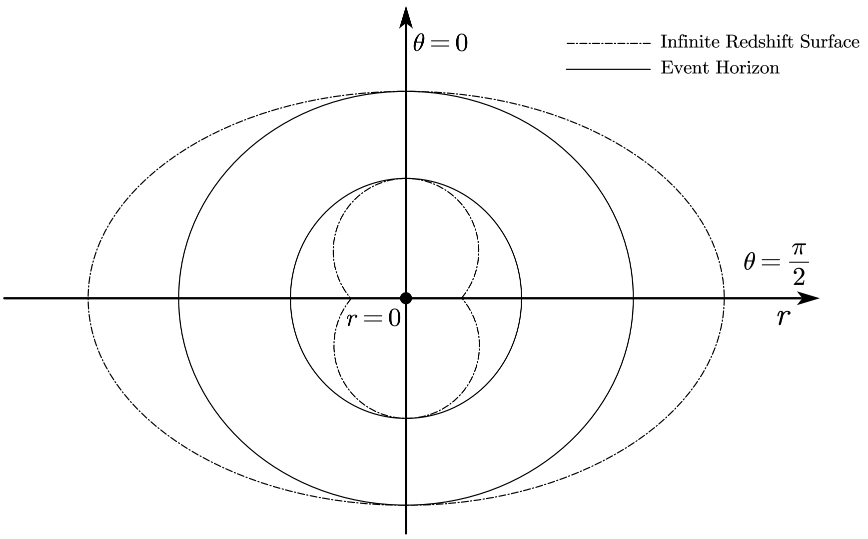

Figure 2. The profile of the Kerr-Newman black hole spacetime

And the velocity decomposition in the θ direction in spacetime

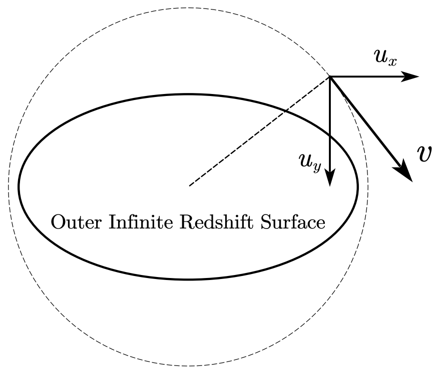

Figure 3. Schematic diagram of velocity decomposition

Using the radial conclusions, and consider the change of u_x and u_y. Because the u_x is in the direction of r increasing, in observation, the infinite redshift surface will contract; the u_y is in the direction of r decreasing, so in observation, the infinite redshift surface will expand. Finally, it can be proved by numerical calculation that the two effects cancel each other, there is no difference in observation between the observer moving in the θ direction and the static observer at infinity.

5. Minimum blueshift radius and intermediate-mass black holes

In the above discussion, a very important concept appeared, the Minimum Blueshift Radius. There is equation (16) as its expression in the most general spacetime. The reason for its appearance is that we study the size of the infinite redshift surface when we escape at the speed of light in an asymptotically flat spacetime.

We should note that, for the static observer at infinity, the constraint between the parameters of the black hole is only $M^2 > Q^2 + a^2$, but the minimum blueshift radius gives a stronger constraint

$$M^2 > 2Q^2 + 4a^2 \tag{19}$$

If we understand the original constraint $M^2 > Q^2 + a^2$ as that the charge and angular momentum of a black hole with a certain mass cannot be too large, so the new relationship reinforces the limitation. It is closely related to the evolution and formation of black holes.

When $Q = 0$ (This is suitable for some ideal neutron stars), the original constraint gives $J < M^2$ (We think that extreme black holes violate thermodynamics, so it can't be equal here), and equation (19) gives $J < M^2/2$. Similarly, when $a = J/M = 0$, the original constraint gives $Q^2 < M^2$, and equation (19) gives $Q^2 < M^2/2$. It makes the constraints for the formation of black holes more stringent, and this is a possible reason why the intermediate-mass black holes are difficult to form[14-16], they're probably carrying too much charge, or rotate too fast. There is no such worry about small black holes and supermassive black holes, the results show that the thermodynamic and quantum mechanical properties of small black holes are very active, and they do not have to be constrained by this constraint, while supermassive black holes tend to rotate very slowly[17,18].

However, the existence of a minimum infinite blueshift radius depends on the dynamics of the object, and it may not affect the black hole stability, so it is only a possible constraint.

6. Discussion within the one-way membrane region

As mentioned above, the discussions up to now have been outside the one-way membrane region, either within the inner infinite redshift surface or outside the outer infinite redshift surface, both normal spacetime. We next explore the case of the one-way membrane region of the Kerr-Newman black hole and treat the R-N black hole as an approximation of the Kerr-Newman black hole at $a = 0$.

Since the nature of the $\mathrm{d}r$ and $\mathrm{d}t$ coordinates of the R-N and Kerr-Newman black holes in the one-way membrane region are exchanged, if we consider the velocity on the hypersurface as the derivative of the space-like coordinate to the time-like coordinate, in the one-way membrane region, we have $\mathrm{d}t/\mathrm{d}r = u$. For the radial coordinate of the Kerr-Newman black hole, there is the squared line element

$$\begin{aligned} \mathrm{d}\chi^2 &= -\left(1 - \frac{2Mr - Q^2}{\rho^2}\right)\mathrm{d}t^2 + \frac{\rho^2}{\Delta}\mathrm{d}r^2 \\ &= \left[\left(-1 + \frac{2Mr - Q^2}{r^2 + a^2\cos^2\theta}\right)u^2 + \frac{r^2 + a^2\cos^2\theta}{r^2 - 2Mr + a^2 + Q^2}\right]\mathrm{d}r^2 \\ &= \gamma^2 \mathrm{d}r^2 \end{aligned} \tag{20}$$

Where $\mathrm{d}\chi$ is the proper displacement of the follow-up observer, and note that the Lorentz transformation of the length is $\mathrm{d}\chi = \gamma \mathrm{d}r$. Equation (20) gives the expression for the Lorentz factor in the one-way membrane region. It is particularly important to note here that the one-way membrane region belongs to the geometry of the black hole and is determined by the event horizon, which is the important geometric object of the black hole, not the infinite redshift surface. Therefore, for observers in the ergosphere, the conclusions in the normal spacetime should still be used instead of the conclusions in the one-way membrane region.

When we consider the case of the equatorial plane $\theta = \pi/2$, the singularity of the numerator of the Lorentz factor will give four solutions

$$r_1 = \frac{Mu - \sqrt{M^2u^2 - (u-1)uQ^2}}{u-1} \quad r_2 = \frac{Mu + \sqrt{M^2u^2 - (u-1)uQ^2}}{u-1}$$
$$r_3 = \frac{Mu - \sqrt{M^2u^2 - (u+1)uQ^2}}{u+1} \quad r_4 = \frac{Mu + \sqrt{M^2u^2 - (u+1)uQ^2}}{u+1} \tag{21}$$

It can be calculated that at $u = \infty$, r_2 and r_4 give

$$\lim_{u\to\infty} r_2 = \lim_{u\to\infty} r_4 = M + \sqrt{M^2 - Q^2} \tag{22}$$

r_1 and r_3 give

$$\lim_{u\to\infty} r_1 = \lim_{u\to\infty} r_3 = M - \sqrt{M^2 - Q^2} \tag{23}$$

Obviously, equations (22) and (23) are the classical inner and outer infinite redshift surfaces. In the one-way membrane region, it seems that the larger the velocity the closer the observations are to normal spacetime, this is because $u = \infty$ in the one-way membrane region is equivalent to $u = 0$ in normal spacetime, so the observation result is equivalent to the result in normal spacetime.

Since $\mathrm{d}r$ in the one-way membrane region is a time-like coordinate with unidirectionality, it can only decrease during the movement of the object from the outer horizon to the inner horizon. Such a motion process is away from the outer infinite redshift surface and close to the inner infinite redshift surface, so in the case of $u > 1$, the outer infinite redshift surface is described by r_2 and the inner infinite redshift surface is described by r_1. The same approach can be taken for the movement from the inner horizon to the outer horizon. Here, a diagram of the infinite redshift surface of the one-way membrane region is given

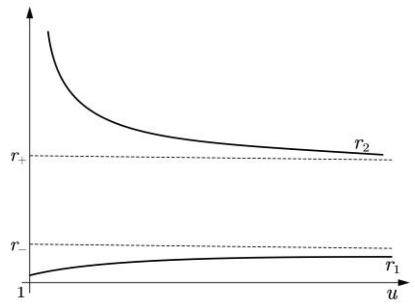

Figure 4. Schematic diagram of the infinite redshift surface in the one-way membrane region

From the above diagram, we see that r_2 is diverging at $u = 1$, it looks as if the outer infinite redshift surface is at infinity. But, since $\mathrm{d}r$ is time-like, infinity here should be understood as unreachable in the sense of time, that is, an object moving from the outer horizon to the inner horizon cannot reach the outer horizon in finite time. In other words, once the object enters the one-way membrane region, it cannot escape and can only move inward. This is obvious and is a proven geometric property of the one-way membrane region[19]. While r_1 gives equations (7) and (15) at $u = 1$, $r_1|_{u=1} = Q^2/2M$, this corresponds to the conclusion that in normal spacetime.

For $\mathrm{d}\theta$ coordinate, which are not time-like in the one-way membrane region, the dynamic effect in this direction are consistent with normal spacetime.

7. Conclusions

In this research, according to the fact that the singularity of the Lorentz transformation leads to the generation of infinite redshift surface, we discussed the dynamic effect of infinite redshift surface. The main results and conclusions are as follows.

1. In the radial direction, the expansion of the infinite redshift surface will be observed when moving towards the black hole, while the contraction will be observed when moving far away from the black hole, and the concept of Minimum Blueshift Radius is defined and studied.
2. When moving in the θ direction, there is no observable dynamic phenomenon on the infinite redshift surface.
3. Due to the appearance of the minimum blueshift radius, new requirements are suggested for three parameters of the black hole (mass, charge, and angular momentum).
4. The dynamic effect of the infinite redshift surface by an observer within the one-way membrane region is investigated.

In the part of complex calculation, some numerical methods are used in this paper.

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